Some Poker Hand Problems

First some basic terminology: A standard poker deck consists of several cards. Each card is identified by a suit (Heart, Spade, Club, Diamond, which I will refer to as H S C D) and a rank which is one of Ace 2 3 4 5 6 7 8 9 10 Jack Queen King (A 2 … 10 J Q K). From this we can immediately determine, from the multiplication principle, that a deck contains 52 cards (but you probably already know that.)

We will be dealing with 5-card "hands". How many hands are possible? This is C(52,5) = 2,598,690. (How did I get that number?) (Answer: I cheated and used the COMBIN function in Excel.)

Two common terms: (1) A *flush* is a hand in which all cards are of the same suit. (2) A *straight* is a hand in which the cards are in order, such as A 2 3 4 5, or 2 3 4 5 6, or …, or 10 J Q K A. Notice that in a straight, the Ace can be at either end.

Example 1: How many hands are straights?

Solution. First we have to choose the starting point of the straight. There are 10 possible starting points. Next, for each of the five cards, we need to choose a suit, of which there are 4. So the answer is $10*4^5$ = 10240. (The probability of being dealt a straight is 10240/2598690 = 0.0039, that is, about 4 tenths of one percent.)

Example 2: How many hands are flushes?

Solution: First we have to choose a suit, of which there are 4. Then, from the 13 cards in that suit, we have to choose 5. So the number of flushes is 4*C(13,5) = 5148. (What is the probability of being dealt a flush?)

Example 3: A "two pair" hand consists of two cards of one rank, two cards of a different rank, and one card of yet another rank. How many hands are "two pair"?

Solution: First we choose the ranks of the two pairs. There are C(13,2) possibilities. Then we have to choose the rank of the fifth card. Two of the 13 are already used, so there are 11 left. Now let's look at one of the pairs. We need two cards of the pair's rank, and there are four cards of that rank, so at this step there are C(4, 2) = 6. Same for the other pair. Finally, we need to choose the suit of the fifth card, for which there are four choices. Putting all this together, we get:

C(13,2)*11*C(4,2)*C(4,2)*4 = ?

Exercise 1: A "one pair" hand consists of two cards of some rank, and three cards of different ranks. How many hands are "one pair"?

Exercise 2: A "full house" contains three cards of one rank and two cards of another rank. How many hands are "full house"?

(See next page for answers.)

Example 3: 123552

Exercise 1: 1098240

Exercise 2: 3744